A STATISTICAL STUDY ON BUFFER-STOCK POLICY

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Summary

The importance of buffer-stock operations for maintenance of price stability is generally recognised. Particularly for foodgrains and for a country like India where their production varies considerably from year to year, these operations are crucial for the health of the general economy. In relation to the buffer stocks the most important questions are those regarding their size and rules of operations, namely, rules governing addition to and releases from the stocks. The estimate of size of buffer stock necessary for success of various buffer stock policies with probability of P=0.90 were brought together. The policies considered envisaged full or partial adjustment for surplus/deficit for different growth rates of demand and supply and for periods of 3,5 and 10 years. If further data give indication of any change in parameters the method adopted in this study may be employed for derivation of results appropriate to the situation.

Introduction

Agriculture in India is often described as a gamble with the monsoon. Though there is considerable expansion in irrigated area during last three decades, vagaries of weather continue to cause large fluctuations in agricultural production. If we consider the trend of production of foodgrains in India since mid-fifties, it is observed that there was accute scarcity of foodgrains all over the country in some years and some parts of the country in many years. There was severe scarcity of foodgrains due to drought in 1965-66 and 1966-67. The production fell short by 17 million tonnes in 1965-66 as compared to the previous year (Sen, S.R., 1967). Such fluctuations in the production of foodgrains cause serious hardship to both, consumer and producer. An important instrument in the hands of the Government for dealing with such a situation is the maintenance of buffer-stocks which are described as stocks held by the Government or by any agency on behalf of the Government for evening out the inter-seasonal fluctuations in supply over the years.

Instability in agricultural production and its behaviour was studied by Swaminathan (1972), Simaika (1974), Reutlinger (1976) and Bigman and Reutlinger (1979). All the authors recognized the importance of buffer stock as a means for stabilizing the supply and price of foodgrains.

For developing the model to deal with the magnitude of buffer stock, the form of the distribution of the foodgrains production as a random variable assumed by the various authors, is different. Ram Saran (1973), Bigman and Reutlinger (1979) assumed that the random variable follows normal distribution, with certain mean and variance Simaika (1974) assumed it to follow the Binomial distribution. This is for making the calculations easy which will be discussed in detail later.

To decide about the quantity to be stored in the buffer-stock, a trend line (compound function) might be fitted to the production and it is suggested that a certain amount of surplus above the trend line should be stored in the stock, and of deficit below the trend line should be taken out from the stock under certain rules. The simplest rule is to put the totality of the surplus in the stock in the surplus years and meet out the total deficit from the stock in the deficit years. But it has been felt that in the surplus years the actual consumption will generally be higher than the expected demand; thus it will not be possible to put the totality of the surplus into stocks while in deficit years, the consumption will be below the expected demand because of higher prices and lower real incomes, it is not necessary to make up all the deficit from stocks. All these cases have been discussed here.

The Model and Method of Calculations: Basically the model for buffer-stock operations has to take into account the demand and supply functions. At the national level demand for cereals will depend mainly on the population which has been steadily growing over the years. Another factor which is expected to affect demand is income. It has been observed from the available data that in India the *per capita real income* has not changed appreciably over the years. Thus demand might be taken to be growing at the rate at which population has been growing or at a slightly higher rate. However, production and consequently supply is affected to a considerable extent by weather and as a result there is a strong random component affecting supply. The situation is therefore well represented by the 'reservoir' model in which the outgoing quantity (represented by demand) is steady whereas the incoming quantity varies substantially over time because of the random component. Simaika (1974) studied the problem of buffer stock on the basis of such a model and the model seems to represent the situation satisfactorily. Thus in the present study two assumptions have been made, one that the demand and supply of cereals were initially equal and have been increasing at the same rate and the second, that the supply is affected by a random component which is also increasing over time and whose magnitude is to be estimated on the basis of actual data. The frequency distribution of the random term also needs to be taken into account in calculation of probabilities of success of stock and allocation policy, success signifying the ability to meet the deficit in bad years. For this purpose it will be reasonable to assume that the random term follows the normal distribution. For the purpose of calculation, however, it may be approximated by a binomial distribution as it is a well-known proposition in statistical theory that a binomial distribution approaches the normal asymptotically.

In postulating the model some more assumptions may be made for reasons given below :

(*i*) Policy is applicable in a compact geographical area implying thereby that the problems of transfer of grain from one part of the area to another need not be considered, nor the possibilities of export from or import into the region envisaged. This is realistic for a country like India where strenuous effort is being made to achieve self-reliance and meeting the deficits in one part of the country from the surplus in another is a regular practice.

(*ii*) Storage capacity is unlimited and storage losses are negligible. In our country the assumption is necessary as we cannot afford to let the limitation of storage capacity stand in the way of stocking of all the surpluses in good years as it can have disastrous consequences in years of short fall. Further the losses are assumed to be negligible as it is the duty of the buffer-stock agency to keep the losses to the minimum by employing the most modern technology.

(*iii*) The buffer stock authority/agency has adequate powers to implement the policy.

The supply function is composed of two parts—a monotonic increasing function of time and a random component increasing in a parallel way. It can be represented by :

$S_N = Ae^{aN} + Xe^{aN}$, where

A : Trend value of production at the initial time.

a : The annual rate of increase in production.

X : A random variable following a probability distribution $F(x)$ which is equal to the probability that $X \leqslant x$ and $dF(x)$ is the probability that X lies between x and $x+dx$. For the case of the computation it is assumed that X follows binomial distribution with the probability function given by

$$P_{(x)} = {}^{16}c_8 + \frac{2x}{\sigma} \left(\tfrac{1}{2}\right)^{16} \qquad \ldots(1)$$

$$x = -4\sigma, \frac{-7\sigma}{2}, \ldots\ldots, \sigma, o, \sigma \ldots, \frac{7\sigma}{2}, 4\sigma$$

The corresponding values of probability are given in Table (2).

N : is the number of years for which the policy is to be framed with $N=o$ at the start of the policy.

Similarly the demand function may be assumed to be of the form $D_N = Ae^{cN}$ where

c is the annual rate of increase in demand, the initial demand that for $N=o$, being assumed equal to the initial supply, *viz.* A. The parameters A, a and σ can be estimated from the data on production for the last 25 years (from 1954-55 to 1977-78). The value of all these parameters are given in Table (1). Buffer Stock Policy is considered under two alternative situation with regard to c, one in which $c=a$ and another in which $c<a$. The situation $c>a$ need not be considered for it is obvious that buffer-stock policy cannot succeed with $c>a$ in which case a situations of chronic scarcity is bound to arise after a period depending upon the initial stock and the difference $(c-a)$.

Some of the other notations used in the study are :—

f_s=the fraction of the surplus in production to be put to the stock and $o \leqslant f_s \leqslant 1$.

f_d=the fraction of the deficit in production to be made up from the stock and $o \leqslant f_d \leqslant 1$.

β_o=the initial stock at the start of the policy.

S'_N=the operating supply in the particular year N. It may be smaller or equal to S_N. Particularly it can be represented as :

TABLE 1

Years	*Production (in 10 thousand tonnnes)*	*Expected Production*	*Difference di*
1954-55	5709	5475	234
55-56	5581	5636	—55
56-57	5830	5802	28
57-58	5475	5973	—498
58-59	6399	6148	251
59-60	6487	6329	158
60-61	6931	6516	416
61-62	7095	6707	388
62-63	6862	6904	—42
63-64	7057	7107	—50
64-65	7694	7317	377
65-66	6240	7532	—1292
66-67	6588	7753	—1165
67-68	8295	7982	313
68-69	8360	8216	144
69-70	8781	8458	323
70-71	9660	8707	953
71-72	9407	8963	444
72-73	8571	9227	—656
73-74	9466	9498	—32
74-75	8981	9778	—797
75-76	10800	10065	735
76-77	9981	10362	—381
77-78	11381	10666	751

σ = 5609, 055 (Thousand Tonnes)
A = 53186 (Thousand Tonnes)
a = 2.94 (per cent)

$$S'_N = Ae^{cN} + f_s xe^{aN} \quad \text{(in surplus years)}$$
$$= Ae^{aN} + xe^{aN} \quad \text{(in deficit years)}$$

D'_N = the operating demand in the particular year N. It can be represented as

$$D'_N = Ae^{cN} \quad \text{(in surplus years)}$$
$$= Ae^{aN} + (1 - f_d)xe^{aN} \quad \text{(in deficit years)}$$

τ_N=effective change in the stock in the particular year N. It is equal to $S'_N-D'_N$. S_N, D_N and τ_N are linear functions of x, in particular τ_N can be represented by:

$$\left.\begin{array}{ll}\overset{+}{\tau}_N=\overset{+}{\lambda}x+\mu & \text{(in surplus years)}\\ \overset{-}{\tau}_N=\overset{-}{\lambda}x+\mu & \text{(in deficit years)}\end{array}\right\}\quad ...(2)$$

$P_N(\beta_0)$=the probability that with an initial stock β_0, the scheme succeeds during N following years.

We start with an initial stock β_0, with $0\leqslant\beta_0,\leqslant\infty$, and

$$\beta_1=\beta_0+(S_1'-D_1')=\beta_0+\tau_1$$
$$\vdots$$
$$\beta_N=\beta_{N-1}+\tau_N$$

For the success of policy $\beta_0, \beta_1,...\beta_N$ must all be greater than or equal to zero.

In derivation of the fundamental recurrence relation, the following steps are involved :

(i)	$\tau_1<-\beta_0$	the policy fails. $P_N(\beta_0)=0$
(ii)	$-\beta_0\leqslant\tau_1\leqslant\mu$	the prob. of success of the policy during N years (deficit) $P_N(\beta_0)=P_{N-1}(\beta_0+\overset{-}{\tau}_1)$
(iii)	$\tau_1\geqslant\mu$	the prob. of success of the policy during N years (surplus) $P_N(\beta_0)=P_{N-1}(\beta_0+\overset{+}{\tau}_1)$

Thus summarising all the above steps, the recurrence relation between P_N and P_{N-1} can be written as :

$$P_N(\beta_0)=\int_{\tau_1=-\beta_0}^{\tau_1=\mu} P_{N-1}\,(\beta_0+\overset{-}{\tau}_1)\,dF(x)+\int_{\tau_1=\mu}^{\infty} P_{N-1}(\beta_0+\overset{+}{\tau}_1)\,dF(x)$$

Replacing $\overset{-}{\tau}_1$ and $\overset{+}{\tau}_1$ by the values $S'_1-D'_1$ (in surplus years and deficit years separately), the equation can be written as :

$$P_N(\beta_0)=\int_{-\beta_0/f_d e^{aN}}^{0} P_{N-1}\,(\beta_0+f_d x e^{aN})\,dF(x)$$
$$+\int_{0}^{\infty} P_{N-1}\,(\beta_0+f_s x e^{aN})\,dF(x)$$

observing that $P_0(\beta_0)=1$ for $\beta_0 \geqslant 0$, the above recurrence formula permits the successive evaluation of $P_1, P_2, \ldots, P_N$.

Now putting

$$\beta_0 = kf\frac{\sigma e^{aN}}{2},$$

where k might take values of 0, 1, 2,...and f equal to f_s or f_d which will be used to simplify the representation, and observing that in the binomial distribution

$$x=\frac{r\sigma}{2}$$

where r takes the values $-8, -7, \ldots, -1, 0, 1, \ldots, 7, 8$ with prob. $P_r={}^{16}c_{8+r}(\frac{1}{2})^{16}$, the recurrence formula becomes

$$P_N\left(kf\frac{\sigma e^{aN}}{2}\right)=\sum_{r=-\text{ integ. } kf/f_d}^{r=-1} P_r P_{N-1}\left| f\frac{\sigma e^{aN}}{2}\left(k+\frac{f_d\, r}{f}\right)\right|$$

$$+\sum_{r=0}^{8} P_r\, P_{N-1}\left| f\frac{\sigma e^{aN}}{2}\left(k+\frac{f_s\, r}{f}\right)\right| \qquad \ldots(3)$$

It can be easily seen that each of the $P_N(\beta)$ is the sum of the seventeen products $P_r\, P_{N-1}(\beta)$, where the $P_r s$ are the seventeen terms of the binomial distribution.

Ultimate formula for the calculation of the initial stocks associated with the probability of success has been derived in the following four cases :

Case I : We assume that $a=c$ and $f_s=f_d=f=1$, the recurrence formula becomes :

$$P_N\left(k\frac{\sigma e^{aN}}{2}\right)=\sum_{r=\text{integ. } k}^{8} P_r P_{N-1}\left|\frac{\sigma e^{aN}}{2}(k+r)\right|$$

If we denote $P_N\left(k\frac{\sigma e^{aN}}{2}\right)$ by $\pi_N(k)$, the above formula becomes

$$\pi_N(k)=\sum_{r=-\text{integ. } k}^{8} P_r\ \pi_{N-1}\ |\ e^a\,(k+r)\ |$$

Since e^a is of the order of 1·02 the values of $\pi_{N-1}\ e^a(k+r)$ are nearly equal to $\pi_{N-1}(k+r)$. Thus :

$$\pi_N(k)=\sum_{r=-\text{integ. } k}^{8} P_r \pi_{N-1} \mid (k+r) \mid \qquad \ldots (4)$$

Case II : We assume that $a=c$ and $f_s=f_d=f=0·75$. Denoting $P_N\left(kf\frac{\sigma e^{aN}}{2}\right)$ by $\pi_N(k)$, the recurrence formula becomes:

$$\pi_N(k)=\sum_{r=-\text{integ. } k}^{8} P_r\ \pi_{N-1} \mid (k+r) \mid$$

which is exactly same as equation (4). Thus there is change in the stock, which is f times that *given in* (4).

TABLE 2

Form of Binomial Distribution	*Binomial Coefficient*	*Probability*
$^{16}c\left(8+\frac{2x}{\sigma}\right)(\frac{1}{2})^{16}$	$^{16}C_{16}=1$	.0000152588
	$^{16}C_{15}=16$	.0002441408
$x=-4\sigma, \frac{-7}{2}\sigma. .0,. ., \frac{7}{2}\sigma, 4\sigma$	$^{16}C_{14}=120$	.0018310560
	$^{16}C_{13}=560$	.0085449280
	$^{16}C_{12}=1820$	.0277710160
	$^{16}C_{11}=4368$	.0666504384
	$^{16}C_{10}=8008$	.1221924704
	$^{16}C_{9}=11440$	.1745606720
	$^{16}C_{8}=12870$	.1963807560

Case III : Here, we assume $a>c$ and $f_s=f_d=f=1$. When the rate of increase of supply and demand are different, the quantity in stock will be submitted to a further change to $Ae^{aN}-Ae^{cN}$. Since the difference $a-c$ is very small (of the order of 10^{-3}), this quantity is approximately equal to $Ae^{aN}(a-c)N$ and for ease of computation will be represented by $\gamma\frac{\sigma}{2}Ne^{aN}$ where $\gamma=\frac{2A}{\sigma}(a-c)$.

The recurrence formula (3) becomes:

$$P_N\left(kf \frac{\sigma e^{aN}}{2}\right) = \sum_{\substack{r=-\text{integ.}\\ kf+\gamma N/f_d}}^{-1} P_r P_{N-1} \left| kf \frac{\sigma e^{aN}}{2} + r f_d \frac{\sigma e^{aN}}{2} + r \frac{\sigma N e^{aN}}{2} \right|$$

$$+ \sum_{r=0}^{8} P_r P_{N-1} \left| kf \frac{\sigma e^{aN}}{2} + r f_s s \frac{\sigma^a e^{aN}}{2} + \gamma \frac{\sigma N e^{aN}}{2} \right|$$

Let ψ be the highest common factor between $f=f_s$, f_d and γ so that $f_s=f_d=f=R\,\psi$. and $\gamma=T\psi$, then the above recurrence relation becomes :

$$P_N\left(KR\,\psi \frac{\sigma e^{aN}}{2}\right) = \sum_{r=-\text{integ.}\frac{KR+TN}{R}}^{8} P_r\, P_{N-1}/\psi \; \frac{\sigma e^{aN}}{2}(KR+\gamma\, R+TN)/$$

Using the transformation of the P's into π's as was used earlier, we get

$$\pi_N(KR) = \sum_{r=-\text{integ.}\frac{KR+TN}{R}}^{8} P_r\, \pi_{N-1}/(KR+\gamma R+TN)/$$

π'_N s are to be calculated for all integral values of KR $(=0, 1, 2, \ldots,)$.

The magnitude of buffer-stock is $KR\,\psi \frac{\sigma e^{aN}}{2}$

Case IV: For $a>c$ and $f_s=f=f_d=0.75$, the formula for the calculation of probability of success remains same. There will be difference in calculation of ψ, T and R. Instead of take $R\,\psi=1$ and $T\psi=\gamma$, we take $R\,\psi=0.75$ and $T\,\psi=\gamma$, here. The stock required for the success of policy to a certain probability level will automatically change.

The results obtained on the basis of these formulae are given at the end in table (3). These results serve to provide us useful guidance in the discussion of policy questions. Such discussion is presented below.

TABLE 3

Summary Table of Results

$A=53186$ (Thousand Tonnes) Prob. of success=.90
$\sigma=5609$ (Thousand Tonnes)

S. No.	*Different conditions*	*Buffer Stock (Thousand Tonnes)*		
		N=3 Years	*N=5 Years*	*N=10 Years*
1.	(*i*) $a=c=.025$ $f_s=f_d=1$	13070	19068	32434
	(*ii*) $a=c=.0294$	13178	19491	33868
2.	(*i*) $a=c=.025$ $f_s=f_d=.75$	9903	14301	24326
	(*ii*) $a=c=.0294$	10063	14618	25401
3.	$a>c$ $f_s=f_d=1$			
	(*i*) $a=.025$			
	$c=.024$ $(a-c=.001)$	12367	18719	31644
	(*ii*) $a=.025$			
	$c=.020$ $(a-c=.005)$	12050	17474	28856
	(*iii*) $a=.03$			
	$c=.02$ $(a-c=.01)$	11379	16371	13622
4.	$a>c$ $f_s=f_d=.75$:			
	(*i*) $a=.025$			
	$c=.024$ $(a-c=.001)$	9628	14004	23639
	(*ii*) $a=.025$			
	$c=.020$ $(a-c=.005)$	8934	12955	15709
	(*iii*) $a=.03$			
	$c=.02$ $(a-c=.01)$	8423	9269	1436

Discussion and Conclusions

It has been found by the analysis of data on production of cereals from 1954-55 to 1977-78 that our cereal production has been increasing during this period at an annual growth of 2.94 per cent and a measure of variation at the begining of the period is given by $\sigma=5.609$ million tonnes. It may be observed that the State and the Central Governments have been making considerable efforts to push ahead the family planning programme and in some areas encouraging results have been obtained. It is hoped that in the eighties the growth rate of production would be closer to 2.0 rather than to 2.5.

Keeping in view the above facts for all the four cases discussed earlier the stocks associated with probability of success have been calculated for different values of a and c (a=.0294, .025, .03 and c=.0294, 025, 024, .02). It is observed from the results obtained by taking $a=c=.0294$ that depending exclusively on domestic production and allowing for seasonal variation, the government would have been in a position to meet its obligation to the consumers fully with a probability of 0.90 for a period of 3 years if it had a stock of 13.2 million tonnes at the beginning of the period. To ensure the same probability of success for 10 years the stock required would have been considerably larger, 33.9 million tonnes. With increase in the period the stock required increases considerably. In this connection it may be noted that even under assumption of constant variance of the random term the desirable buffer stock for a given level of probability of success would have increased proportionately to $\sqrt{N}$. With the assumption of increasing variance the increase in the desirable buffer stock is naturally even more rapid. It is clear that the Government was not in a position to build up such stocks and it has to resort to large scale imports in the following period. The more practical strategy that might be recommended in such a situation is, therefore, to maintain stocks adequate to cover a three year period. Depending on the magnitude of the shortfall in a particular year-government may consider imports to make up the deficit. The government will have 3 years at their disposal for this operation. It will be remembered that the buffer stock is proportional to σ and this parameter is increasing over years at the rate of e^{aN}. Extending this result to the present period, say, 1981-82, would involve considerable increase in the size of the desirable buffer-stock.

To bring the result within the range of practical achievement it would be necessary to reduce the level of probability or aim at a shorter period. We might therefore consider the impact of change in certain parameters on the size of desirable buffer stock. The model with $f_s=f_d=0.75$ indicates the effect of such a change. It is seen from the figures that this reduces the required buffer stock also to about 3/4th of the level indicated for $f=1$, making it easier and more practicable.

The conclusions revealed from the table (3) seem to be quite reasonable. It should still be noted that further analysis of data on supply and demand from year to year is necessary to observe whether the parameters of the model are undergoing any change. From this angle the growth rate of production as well as population should be under constant watch. If, as a result of modern technology (irrigation, fertilizer, better varities of crops etc.), the random variation decreases, that is, the stability of production increases the size of the desirable buffer stock can come down. One corollary of the present model is that the size of desirable buffer stock goes on increasing exponentially from year to year. It is possible that this may not hold and in fact the relative (random) variation in output may come down in future in which case we may not be required to keep buffer stock beyond certain limit and any surplus foodgrain available over the limit could be exported or diverted to uses other than direct human consumption. In any case, the present investigation presents a methodology, which may help in taking policy decisions in relation to buffer stock.

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